Name and Surname : ....

Grade/Class : 12/.........   **Mathematics Teacher** : ....

Hudson Park High School

GRADE 12

MATHEMATICS

Paper 1

Marks : ____ / 150

Time : 3 hours   Date : 3 June 2016

Examiner : SLT   Moderator(s) : SLK

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB**
   - **Start each QUESTION at the *top of a page.***
   - **Leave *2 lines* open between each of your answers.**
4. **NB** **Fill in the details requested on the front of this Question Paper and hand in your submission in the following manner :**
   - **Question Paper (on top)**
   - **Answer Pages (below, in order)**

   Do not staple the Question Paper and Answer Pages together.
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

QUESTION 1 [36 marks]

1.1. Solve for x :

1.1.1. $x^2 = 5x$ — 3

1.1.2. $3x^2 - 4x - 12 = 0$ — 3

1.1.3. $10x^{-\frac{2}{3}} + 8x^{-\frac{4}{3}} = 3$ — 6

1.1.4. $0 \leq -x(6x+5) + 4$ — 3

1.1.5. $\dfrac{\sqrt{x}\,(2-x)}{2^x(x-1)} \geq 0$ — 2 (17)

1.2. Given : $6x^2 - 3y = 11x + 10$
$\frac{1}{3}x - y = \frac{16}{3}$

1.2.1. Solve for x and y. — 6

1.2.2. Interpret your answer to (1.2.1.) graphically. — 2 (8)

1.3. Simplify fully : $\dfrac{2^{2015}}{2^{2017} - 3.2^{2012}}$ (3)

1.4. A quadratic equation was solved and it's roots were found to be :

$$x = \frac{3 \pm \sqrt{21-4k}}{5}$$

where $k \in \mathbb{N}_0$. Determine the value(s) of k for which the roots of the quadratic equation will be rational. (2)

1.5. The graph of f has the following equation :

$$y = x + \frac{1}{x}$$

where $x \in \mathbb{R}$ and $x \neq 0$.

1.5.1. Write the given equation in standard form. — 1

1.5.2. Now, determine the discriminant (Δ) of the equation in (1.5.1.). — 2

1.5.3. Hence, determine the range of f. — 3 (6)

QUESTION 2 [28 marks]

2.1. The 10th, 11th and 12th terms of an arithmetic sequence are :

$2x + 3 ; 4x + 10 ; 10x - 3$

2.1.1. Calculate the value of x, showing that it will be 5. 2

2.1.2. Hence, determine T_{10} and T_{11}. 2

2.1.3. Now, calculate T_{500}. 4 (8)

2.2. Evaluate : $\sum_{k=7}^{95}(3-5k)$ (5)

2.3. Given : $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \cdots$

2.3.1. Calculate n if $S_\infty - S_n = \frac{1024}{59049}$ 7

2.3.2. You are given a tree of height 1,5 m as a gift. You plant it immediately. Each year you monitor it's growth and tabulate your results :

Year	Growth (in metres)
1	0,75
2	0,5
3	$0,\dot{3}$

What is the maximum height the tree will grow to ? 2 (9)

2.4. Given : $\frac{4}{16} ; \frac{-4}{8} ; \frac{-18}{4} ; \frac{-38}{2} ; \cdots$

Determine an expression for the n-th term of the sequence, T_n. (6)

QUESTION 3 [19 marks]

3.1. Given : $f(x) = -2^{x-1} + 8$

3.1.1. Sketch the graph of f, showing all relevant details on your diagram. 4

3.1.2. State the range of f. 1

3.1.3. Calculate the average gradient of f between $x = -1$ and $x = 1$. 3

3.1.4. f is reflected in the x-axis to become g. Determine the equation of g in y-form. 2

3.1.5. If $h(x) = -16.2^{x-1} + 10$, give a detailed description of the tansformation of f to h. 3 (13)

3.2. Given : $i(x) = \log_{\frac{1}{4}} x$

3.2.1. Sketch a rough graph of i, showing all relevant details on your diagram. 2

3.2.2. Solve for x : $\log_{\frac{1}{4}} x = 3$ 2

3.2.3. Hence, use your graph to solve for x : $\log_{\frac{1}{4}} x \geq 3$ 2 (6)

QUESTION 4 [10 marks]

4.1 Given : $f(x) = -\dfrac{8}{x+2} + 1$

4.1.1. Sketch the graph of f, showing all relevant details on your diagram. 5

4.1.2. f is reflected in a certain line to become g, where

$$g(x) = \frac{8}{x+2} + 1$$

State the possible equation(s) of that line. 2

4.1.3. If f is moved 5 units to the left, what will the new equation of f be ? 1 (8)

4.2. For $h(x) = \dfrac{5}{x+p} + q$ it is known that :

- the domain is : $x \in \mathbb{R}, x \neq 4$
- one of the axes of symmetry is : $y = -x + 7$

Determine the value of q. (2)

QUESTION 5 [17 marks]

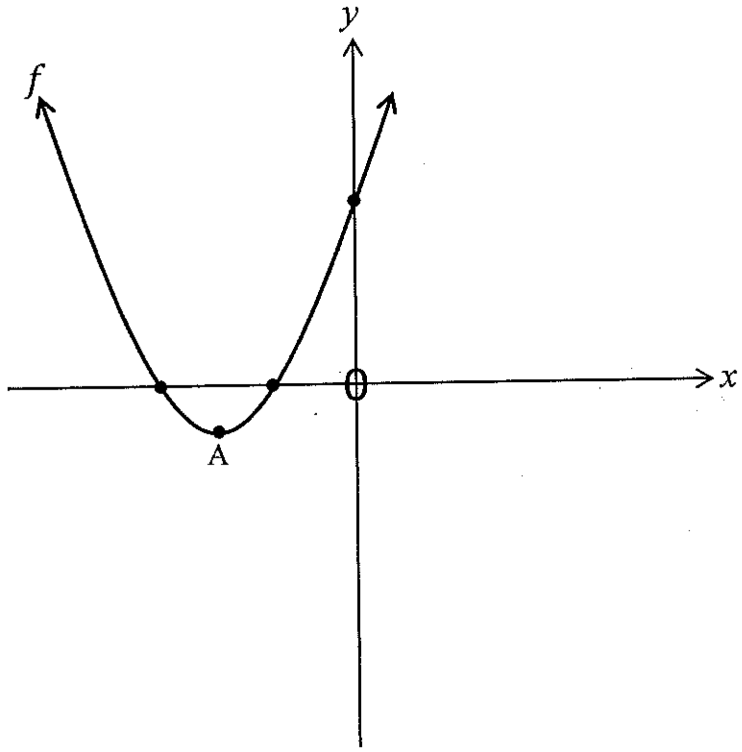
USE THE ANSWER SHEET PROVIDED

5.1. The following details are known about $g(x) = -2x^2 + bx + c$:

- the axis of symmetry is : $x = 3$
- $g(-4) = 0$

Calculate the values of b and c. (4)

5.2. A sketch of $f(x) = \frac{1}{4}(x+5)^2 - 1$ is shown below, where A is the turning point of f :



5.2.1 ~~5.1.~~ For f, determine the y-intercept, x-intercepts and coordinates of A and fill them in on the sketch. 4

5.2.2. ~~5.2.~~ On the same set of axes as f , sketch f^{-1}, the inverse of f. The intercepts and turning point of f^{-1} must be clearly labelled. 3

5.2.3. ~~5.3.~~ Determine the equation of f^{-1}, the inverse of f, in y-form. 4

5.2.4. ~~5.4.~~ f^{-1} is not a function.

5.2.4.1 ~~5.4.1.~~ Give a reason for this, with reference to f. 1

5.2.4.2 ~~5.4.2.~~ State one way in which the domain of f be restricted so that f^{-1} would be a function. 1 2 (13)

QUESTION 6 [12 marks]

6.1. How many full years will it take an investment that is earning 6 % interest per annum compounded monthly, to double in value ? (4)

6.2. A vehicle depreciates ***by*** R 30 000 over a period of 5 years when the rate of depreciation, as calculated on the reducing balance method, is 7 % per annum. Calculate the initial value of the vehicle. (4)

6.3. Convert a nominal interest rate of 6 % per annum compounded monthly, to a nominal interest rate per annum compounded half-yearly. (4)

QUESTION 7 [8 marks]

7.1. When $f(x) = -2x^3 + ax^2 + 4$ is divided by $(x + 3)$ the remainder is -14. Calculate the value of a. (3)

7.2. Given : $f(x) = 6x^3 - 2x^2 + x + 35$

7.2.1. Use the factor theorem to show that $(3x + 5)$ is a factor of f. 2

7.2.2. Hence, determine the other (quadratic) factor of f. 3 (5)

QUESTION 8 [20 marks]

8.1. If $f(x) = \frac{3}{x} - 1$, determine $f'(x)$ from first principles. (6)

8.2. Determine :

8.2.1. $\frac{dy}{dx}$, if $y = \frac{x^2 + 5}{4 . \sqrt[3]{x}}$ 4

8.2.2. $f'(x)$, if $f(x) = x^{\frac{1}{2}}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$ 3

8.2.3. $D_x\left[\frac{8x^3 - 27}{2x - 3}\right]$ 2 (9)

8.3. The tangent to $f(x) = ax^2 + bx + 5$ at the point $(-2 ; 9)$ is perpendicular to the line $7y - 2x + 21 = 0$.
Calculate the values of a and b. (5)

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}\ ;\ r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Detach this page from your question paper and staple it, in order, with your foolscap answers.

Name and Surname : ....

ANSWER PAGE FOR QUESTION 5

5.1.

5.2.